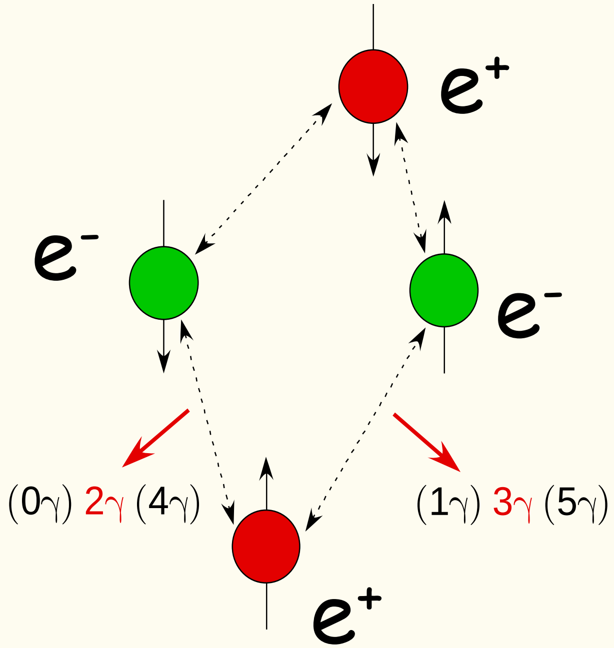
# Dipole excitation of dipositronium




Few-Body Problems in Physics

Bonn, August 31, 2009


Andrzej Czarnecki  University of Alberta

# Outline

Positronium, its ion, and the molecule

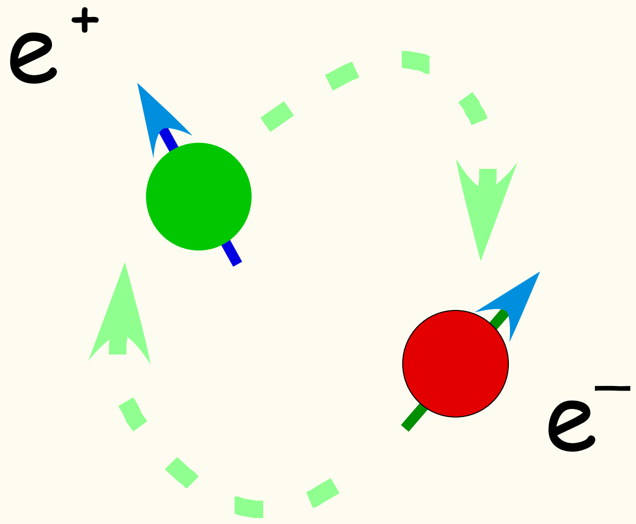
1951 1981 2007

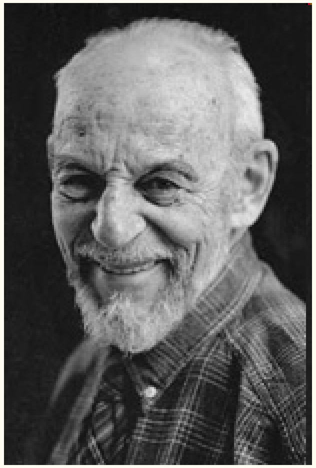
Hyperfine splitting in Ps: new measurement

$Ps^-$ ion: theoretical prediction for the decay rate. Comparison with experiment.

Molecule: spectrum. Dipole transition vs. annihilation.

# Positronium



Martin Deutsch
1917 - 2002

Very similar to hydrogen, except
- no hadronic nucleus
- annihilation
- reduced mass reduced $m_e \rightarrow \frac{m_e}{2}$

Binding: 0.25 a.u.

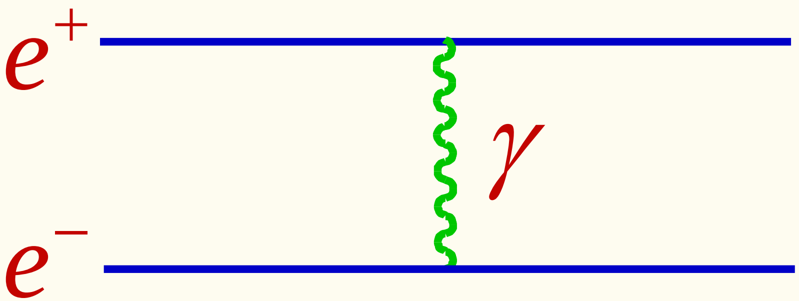
Two spin states:
singlet (para-Ps; short-lived, 0.1 ns)
triplet (ortho-Ps; long-lived, ~150 ns)

All properties can be described by QED, using one parameter: $\alpha = \frac{1}{137.036}$

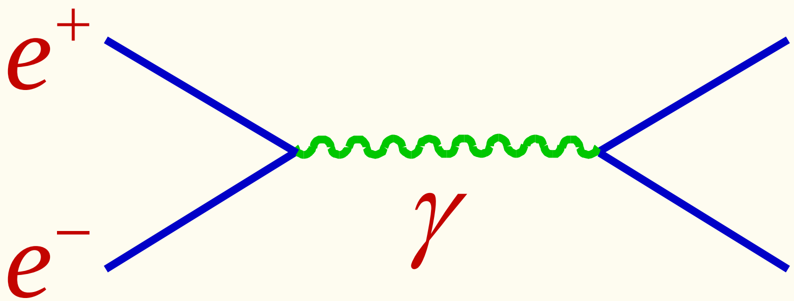
# Positronium spectrum: discrepancy with QED

Tree-level QED prediction for the hyperfine splitting (HFS)

Recoil effect




Annihilation effect

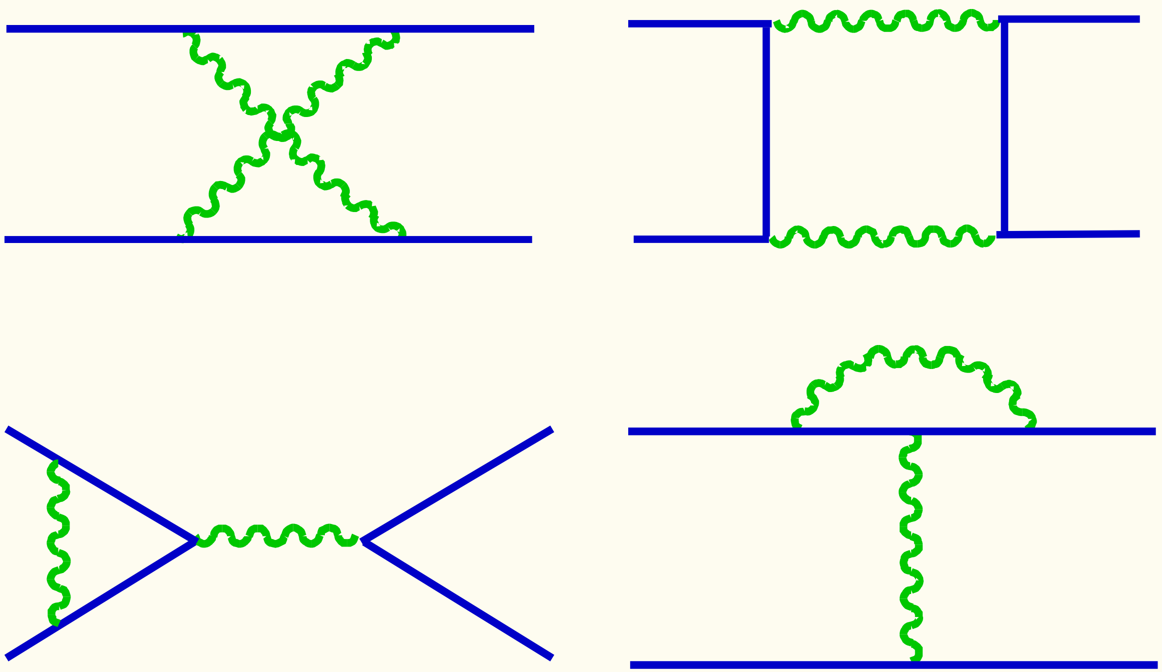



$$\gamma_\mu \otimes \gamma^\mu \to 1 \otimes 1 + \sigma \otimes \sigma$$

$$\Delta\nu_{\mathrm{HFS}} = \frac{7}{12} m_e \alpha^4 \simeq 204 \text{ GHz}$$

# Quantum corrections to the HFS: one-loop

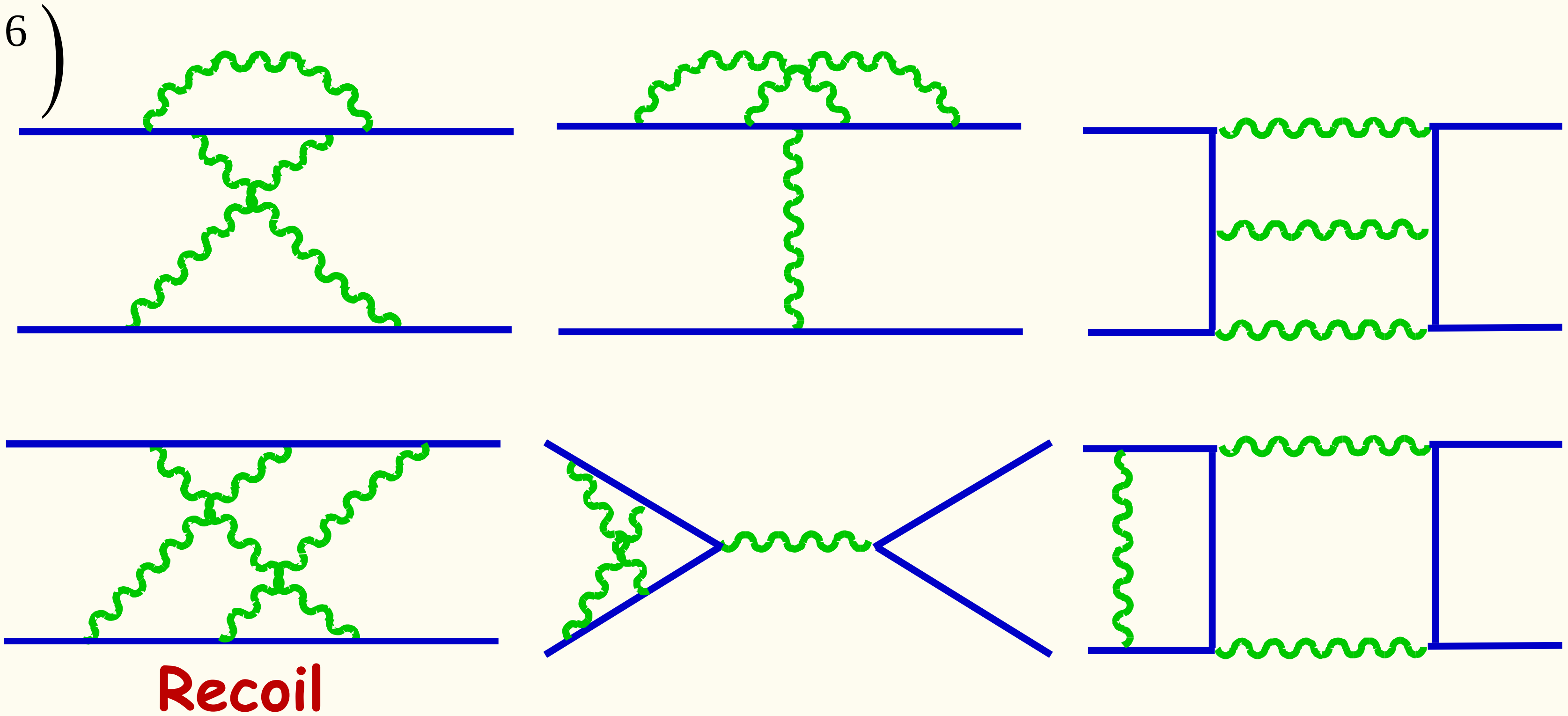
$O\left(\alpha^5\right)$



$$-\frac{m_e \alpha^5}{\pi}\left(\frac{8}{9}+\frac{\ln 2}{2}\right) \simeq -1005.5 \text{ MHz} \rightarrow -0.5\%$$

# Quantum corrections to the HFS: two-loop

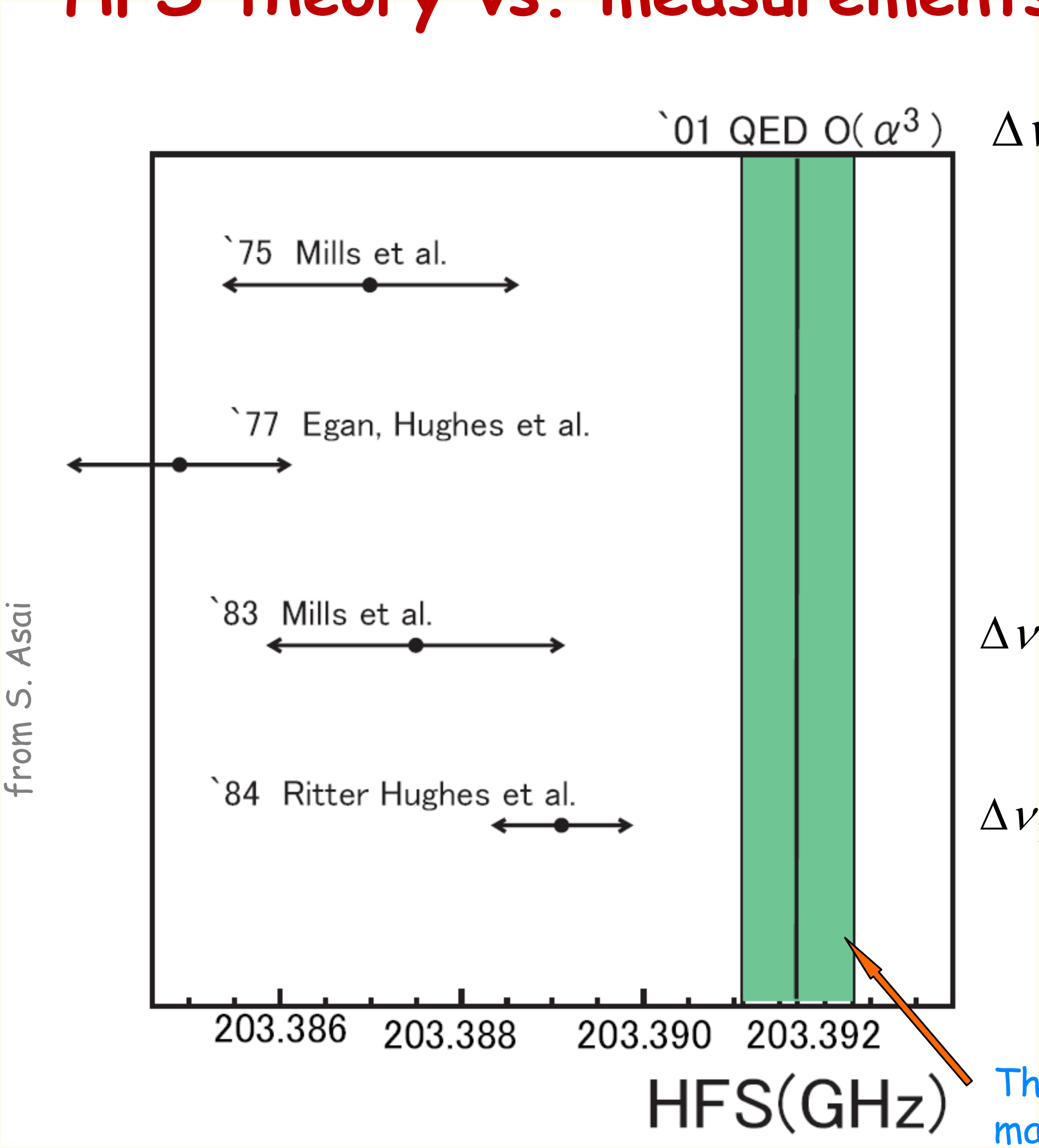
$O\left(\alpha^{6}\right)$




$$\frac{m_e\alpha^6}{\pi^2}\left[\frac{1367}{648}-\frac{5197}{3456}\pi^2+\left(\frac{1}{2}+\frac{221}{144}\pi^2\right)\ln 2-\frac{53}{32}\zeta(3)+\frac{5}{24}\pi^2\ln\frac{1}{\alpha}\right]$$

$\simeq 11.8$ MHz $\rightarrow$ 0.006% (Experimental error $\simeq 0.7$ MHz)

# HFS theory vs. measurements




$$\Delta\nu^{\text{theory}} = 203391.69(16)\ \text{MHz}$$

3.4σ off!

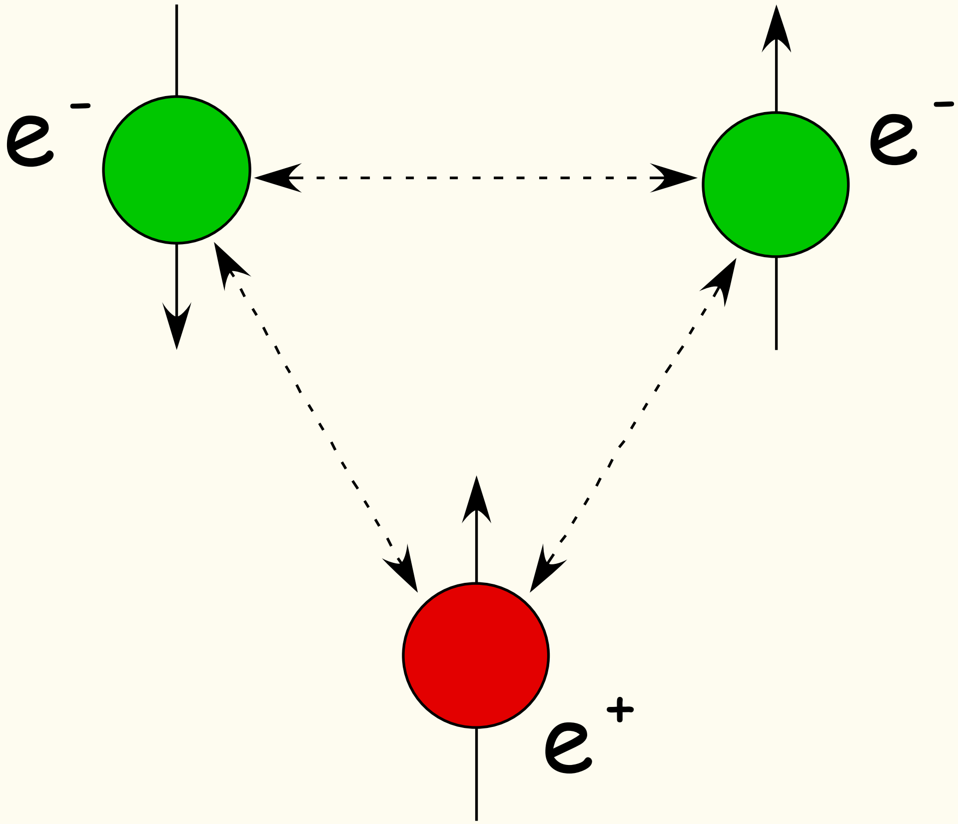
$$\Delta\nu^{\text{exp}}_{\text{HFS}} = 203387.5(1.6)\ \text{MHz}$$

$$\Delta\nu^{\text{exp}}_{\text{HFS}} = 203389.10(74)\ \text{MHz}$$

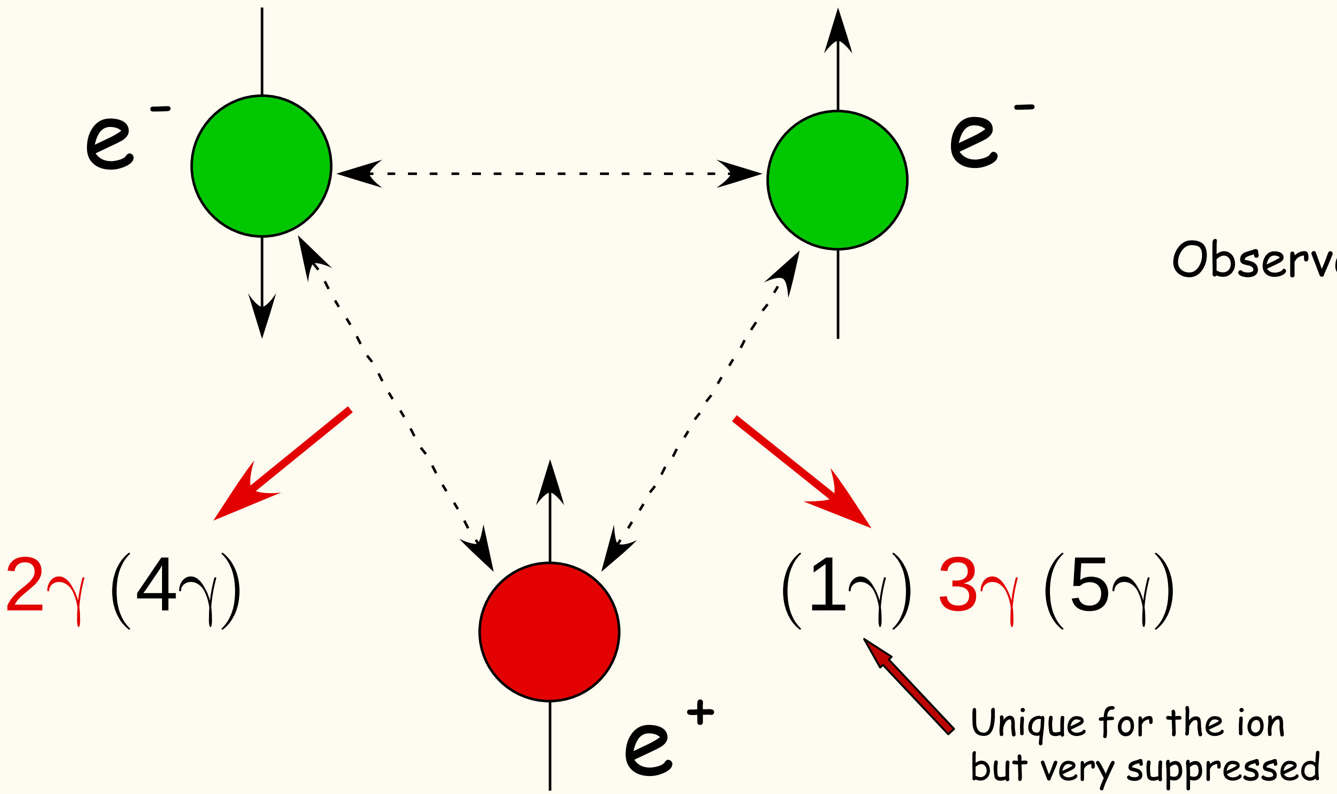
Theory error magnified 4X

# Positronium ion: predicted in 1946

John Wheeler
(1911-2008)



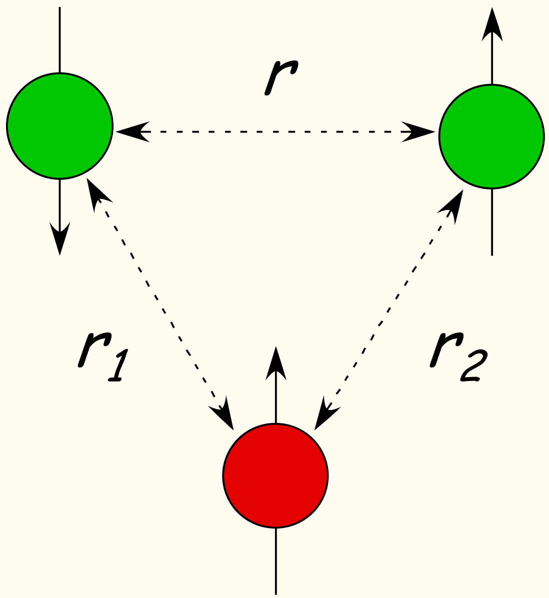
# Positronium ion: a new test of bound-state QED




Observed 1981 Mills

New positronium-ion source
at the FRM II reactor in Munich:
measurements of branching ratios.

# Theory of the Ps ion: the wavefunction




$$\psi(\vec{r}_1, \vec{r}_2) = \frac{\uparrow\downarrow - \downarrow\uparrow}{\sqrt{2}} \phi(r_1, r_2, r)$$

The wave function is not known analytically,
but can be found using the variational method.

The ion resembles a positronium "shell"
and a loosely-bound electron.
Now we can easily estimate the dominant decay channels.

# Recent measurement of the ion decay rate

63401 (2006) PHYSICAL REVIEW LETTERS week
17 FEBRU

## Measurement of the Decay Rate of the Negative Ion of Positronium ($Ps^-$)

Frank Fleischer,[1,*] Kai Degreif,[1] Gerald Gwinner,[1,†] Michael Lestinsky,[1]
Vitaly Liechtenstein,[2] Florian Plenge,[1] and Dirk Schwalm[1]
[1]*Max-Planck-Institut für Kernphysik, Heidelberg, Germany*
[2]*Kurchatov Institute, Moscow, Russia*
(Received 28 November 2005; published 13 February 2006)

A new determination of the decay rate of the negative ion of positronium ($Ps^-$), using a beam-foil method and a stripping-based detection technique, is reported. The measured result of $\Gamma = 2.089(15)\ \mathrm{ns}^{-1}$ is a factor of 6 more precise than the previous experimental value of $\Gamma = 2.09(9)\ \mathrm{ns}^{-1}$, and is in excellent agreement with the theoretical value of $\Gamma = 2.086(6)\ \mathrm{ns}^{-1}$.

$$\Gamma_{\exp}\left(Ps^-\right) = 2089(15)\,\mu s^{-1}$$

$$7\cdot 10^{-3}$$

(factor 4-5
improvement
expected)

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$$\Gamma_{\mathrm{exp}}\left(Ps^-\right) = 2089(15)\,\mu s^{-1}$$

We wanted to improve this

$$7 \cdot 10^{-3}$$

(factor 4-5 improvement expected)

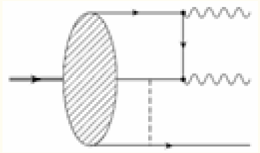
# Positronium ion decay: refinements

Corrections O(α)

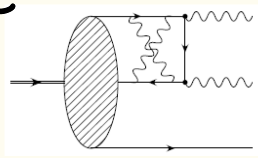
single hard photon loops

Corrections O(α²); challenge: divergences → $\ln\alpha$

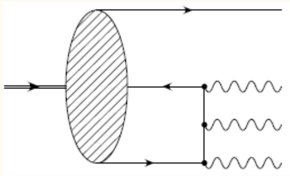
soft
- O(k²) corrections to the amplitude M
- Breit hamiltonian → correction to $\phi$ (r=0)



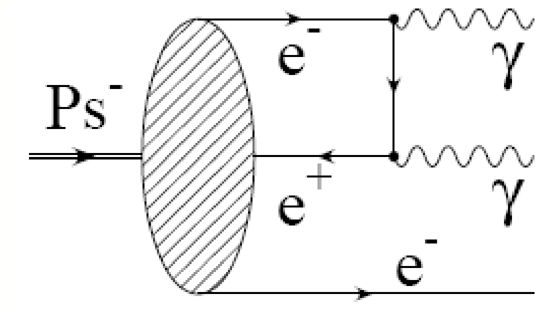
hard
- Short-distance two-loop photon exchange



- Real photon radiation



# Decay rate prediction



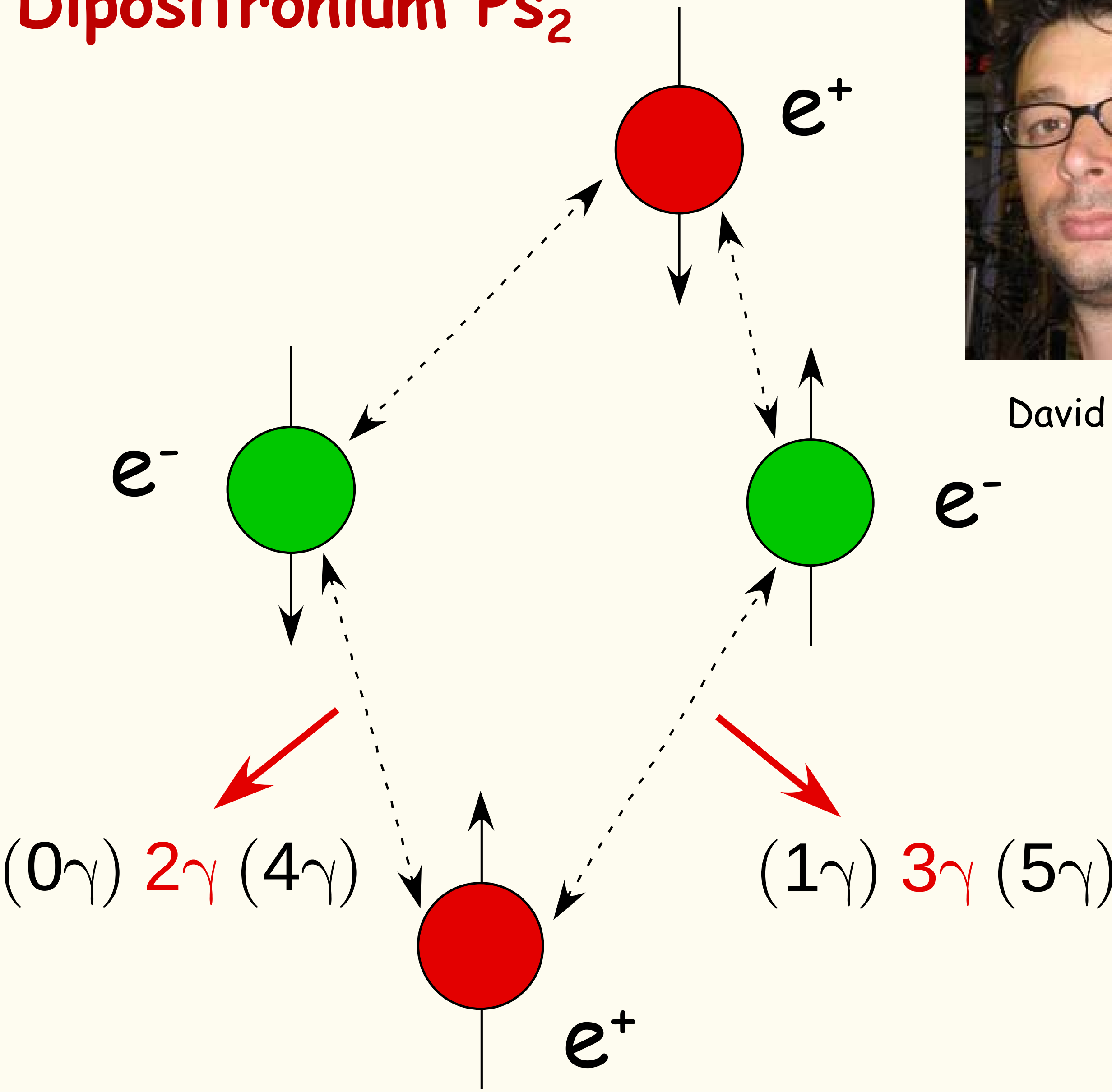

+ corrections

$$\Gamma(\mathrm{Ps}^-) = 2\pi \frac{\alpha^5 m_e c^2}{\hbar}(1 + C)\left\langle \delta^3(r_{12}) \right\rangle$$

$$\Gamma\left(\mathrm{Ps}^-\right) = 2.087\,963\left(12\right)\mathrm{ns}^{-1}$$

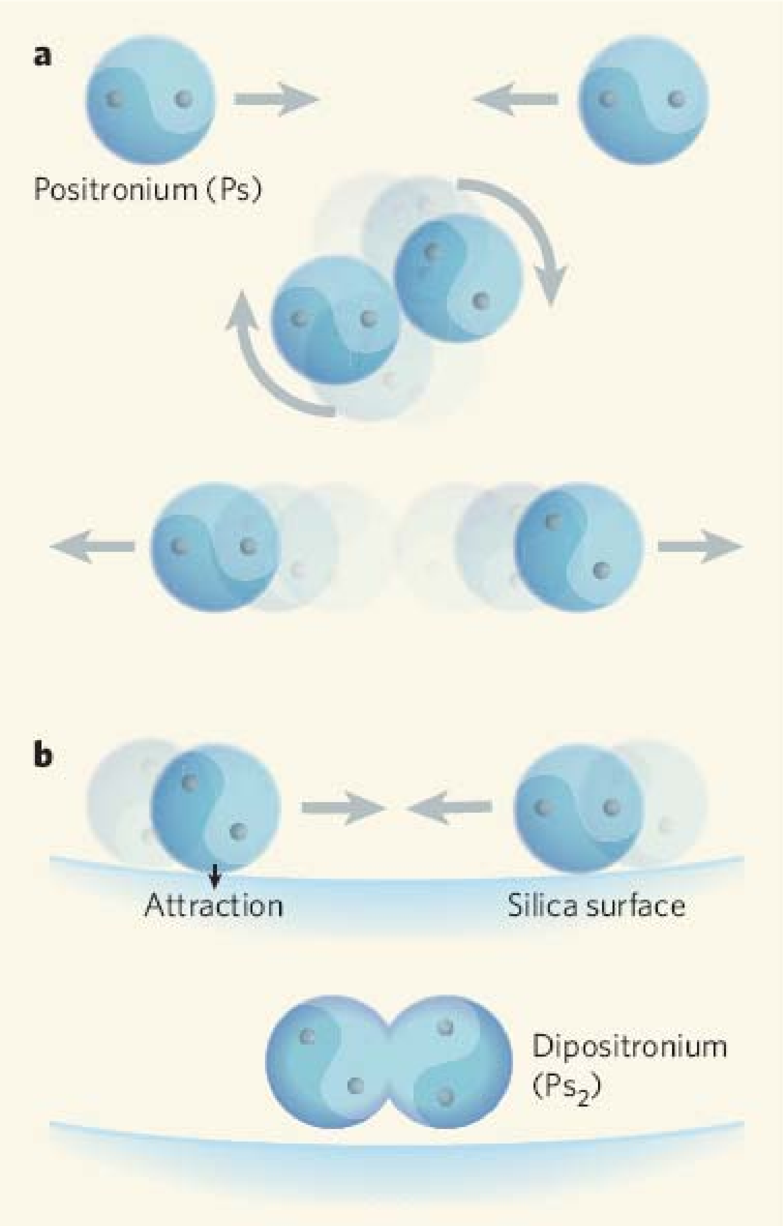
with M. Puchalski and S. Karshenboim
PRL **99**, 203401 (2007)

# Dipositronium $Ps_2$




David Cassidy Allen Mills

# Discovery of dipositronium 2007



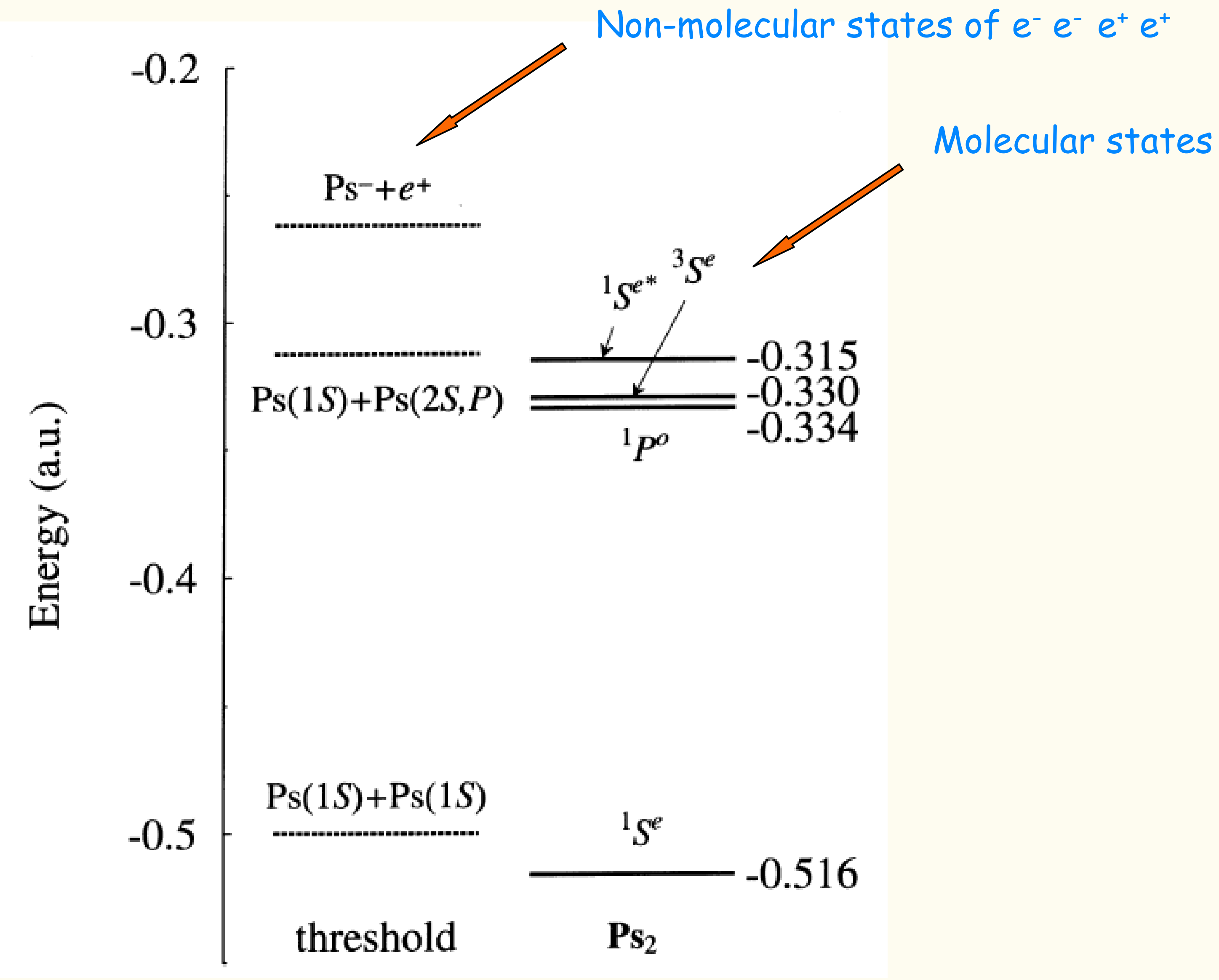

Molecule formation kills long-lived positronia.

At higher temperature, fewer atoms on the surface, fewer molecules formed.

Indeed: at high-T, more long-lived positronia observed.

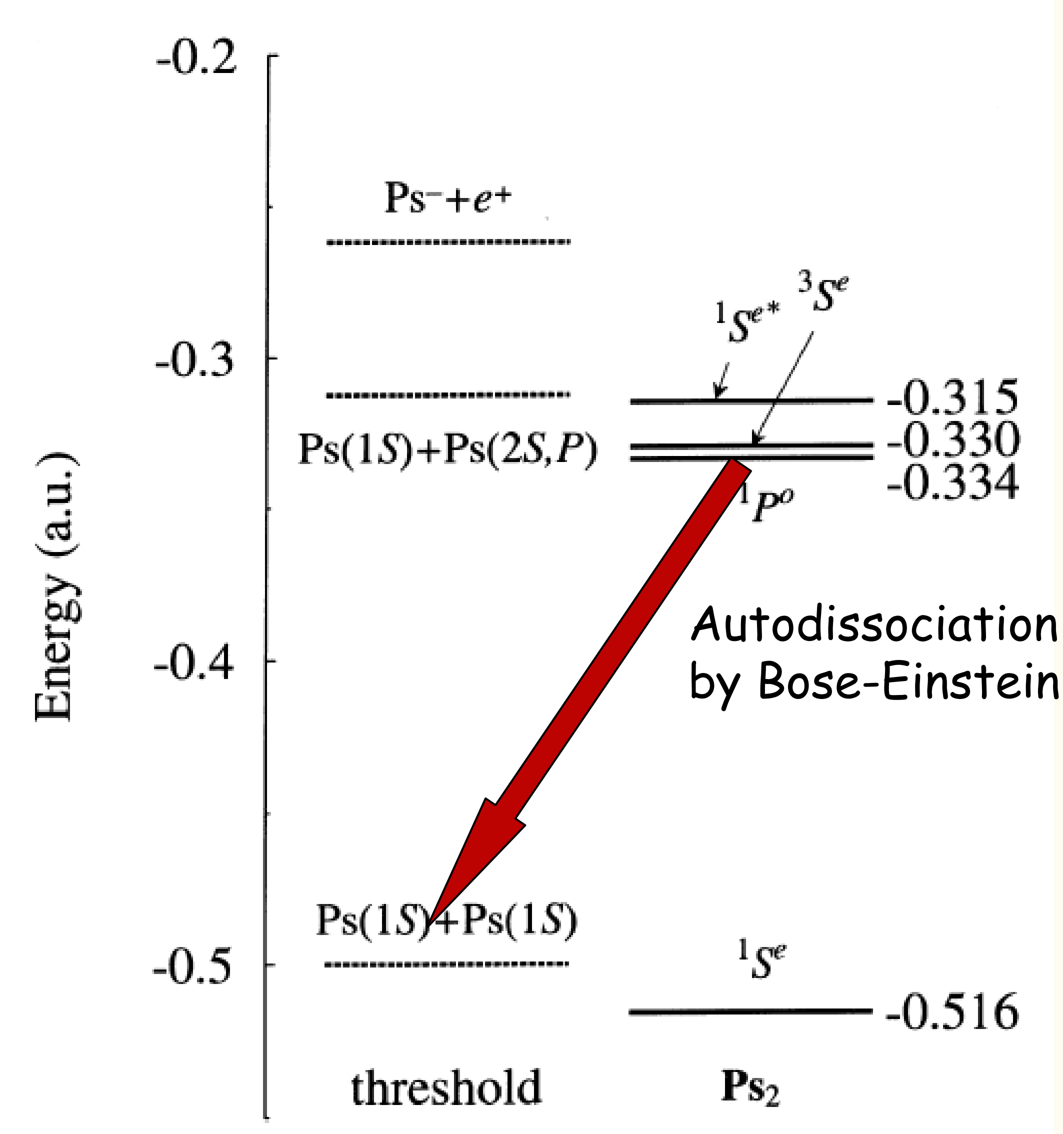
Cassidy & Mills, Nature 2007

# Spectrum of the molecule $Ps_2$




From Suzuki & Usukura, 2000

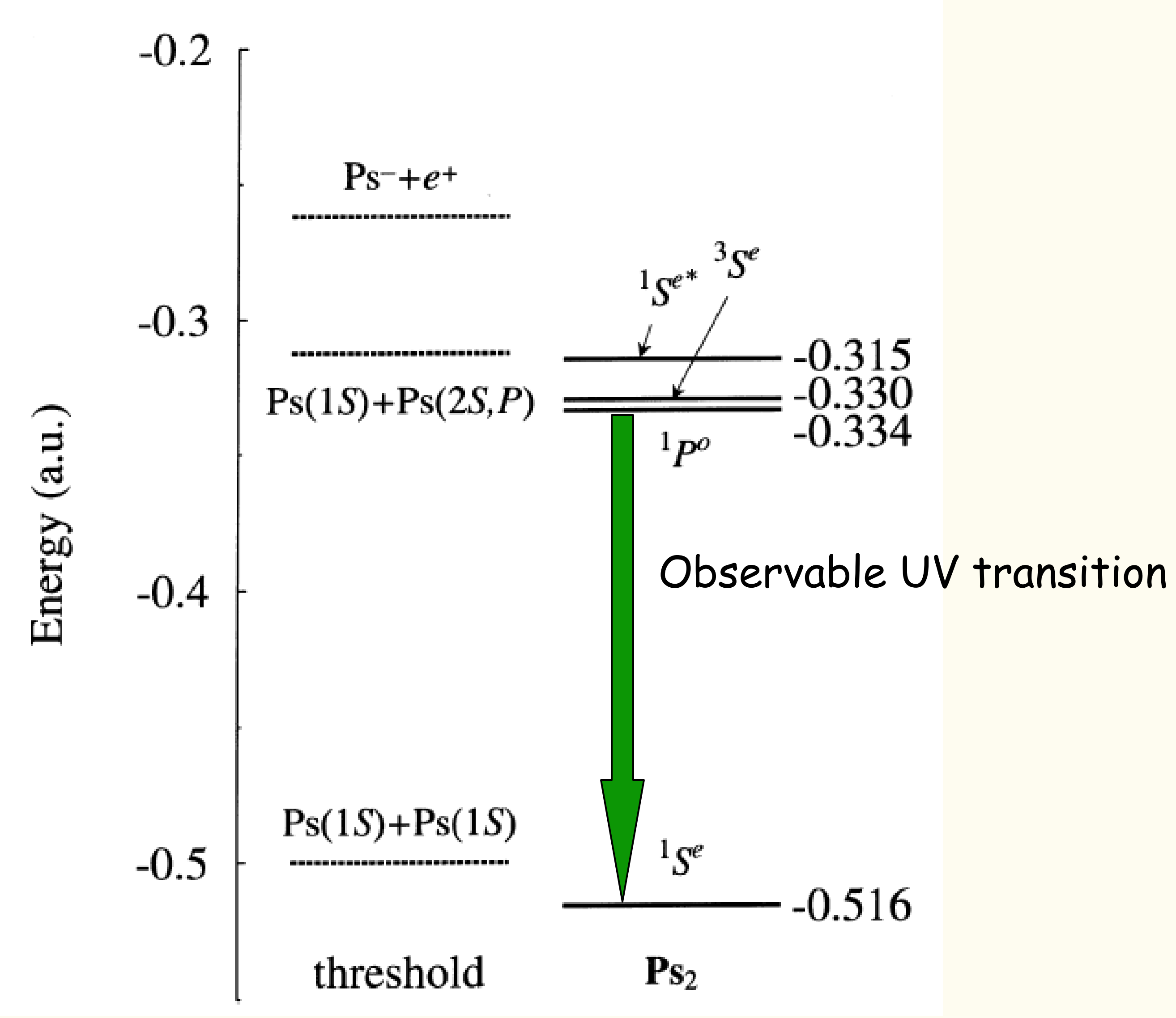
# A direct signal of the molecule: transition line.




Autodissociation forbidden by Bose-Einstein statistics

From Suzuki & Usukura, 2000

# A direct signal of the molecule: transition line.




From Suzuki & Usukura, 2000

with Puchalski,
PRL 101, 183001 (2008)

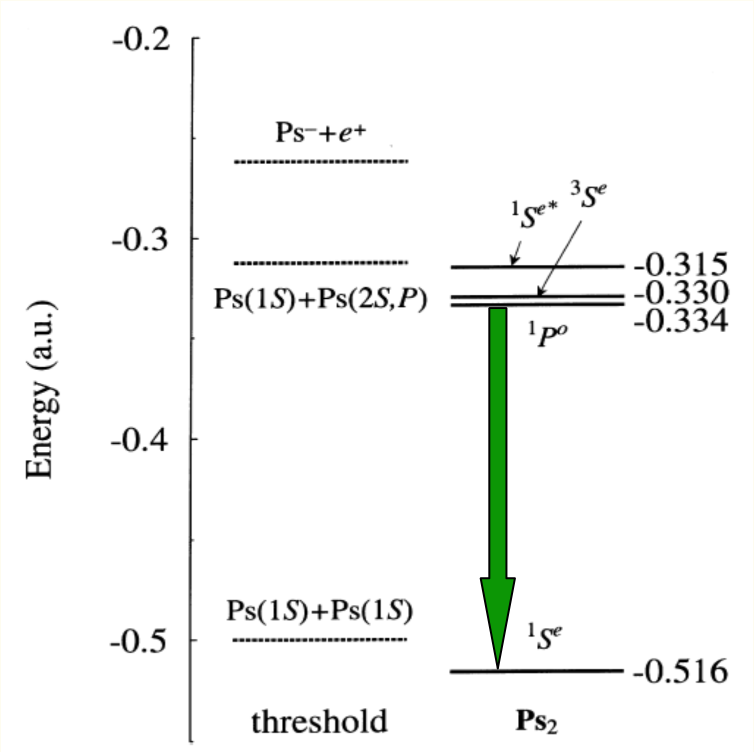
# Questions about this transition:

What is its accurate energy?

$$\Delta E = E_P - E_S = 0.1815867(8)\,\text{a.u.}$$
$$\simeq 4.9\,\text{eV}$$

Similar to atomic positronium, but softer (dielectric effect?):

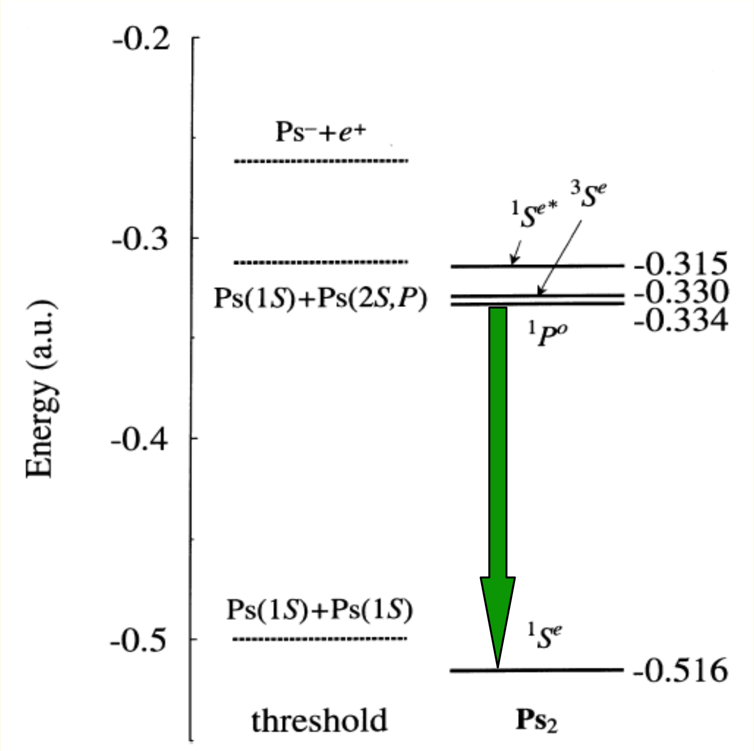
$$E_P - E_S = \frac{3}{4}\times\frac{1}{4}\,\text{a.u.} = 0.1875\,\text{a.u.}$$



with Puchalski,
PRL 101, 183001 (2008)

# Questions about this transition:

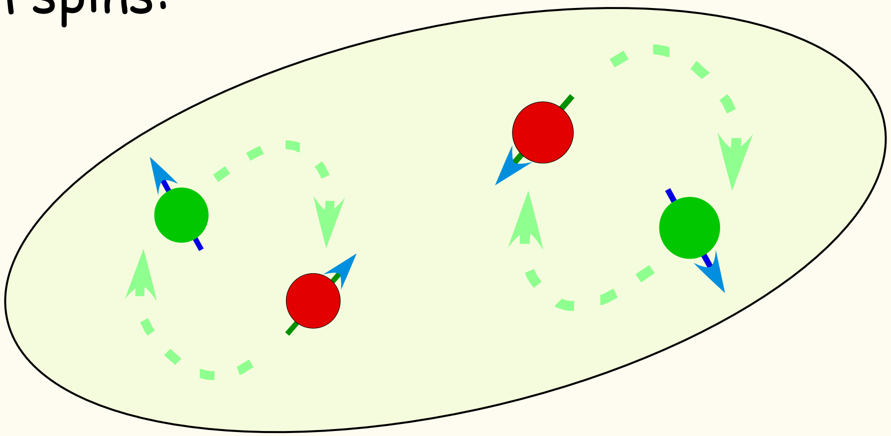
How often does radiative transition appear (before annihilation)?




$$\mathrm{BR}\left(P \to S\right) = \frac{\Gamma_{\mathrm{dip}}\left(P \to S\right)}{\Gamma_{\mathrm{annih}}\left(P\right) + \Gamma_{\mathrm{dip}}\left(P \to S\right)} = 0.191(2)$$

# Competition: dipole transition vs. annihilation

S-state annihilates quite rapidly.
Assume it consists of two weakly-interacting Ps atoms, with random spins:



There is a para-positronium pair with probability $2_{\text{pairs}} \cdot \frac{1}{4_{\text{spin states}}} = \frac{1}{2}$

The decay rate: $\Gamma_S \simeq \frac{1}{2_{\text{Ps probability}}} \cdot \frac{1}{\tau_{\text{pPs}}} = \frac{1}{0.25\,\text{ns}}$

P-state: half of this rate: $\Gamma_{\text{annih}} = \frac{1}{0.5\,\text{ns}}$

# Competition: dipole transition vs. annihilation

In an isolated P-excited Ps atom lives ~3.2 ns;
In $Ps_2$ the decay is about twice faster
(and the same as in atomic hydrogen)

$$d_{\text{atom}} = \left\langle \mathrm{S} \middle| \vec{d} \middle| \mathrm{P} \right\rangle$$

$$d_{\text{molecule}} = \left\langle \mathrm{SS} \middle| \vec{d} \middle| \frac{\mathrm{PS}+\mathrm{SP}}{\sqrt{2}} \right\rangle = \sqrt{2}\, d_{\text{atom}}$$

$$\Gamma_{\text{dip}}(P \to S) = \left(\sqrt{2}\right)^2 \Gamma_{\text{dip}}(\text{atomic Ps}) = \frac{1}{1.6\ \text{ns}}$$

Branching ratio

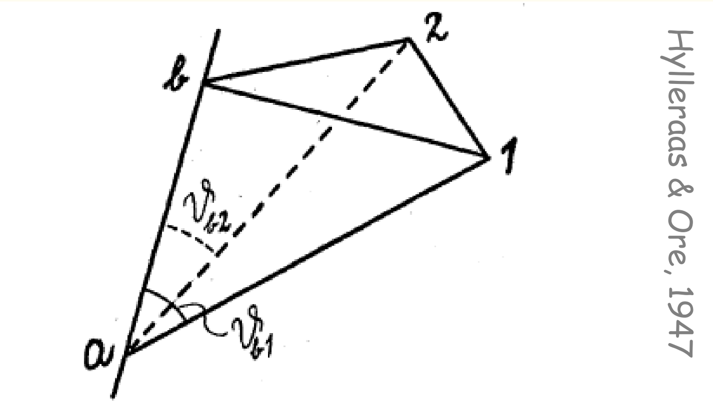
$$BR(\mathrm{P} \to \mathrm{S}) = \frac{\Gamma_{\text{dip}}(P \to S)}{\Gamma_{\text{annih}}(P) + \Gamma_{\text{dip}}(P \to S)} \sim \frac{\frac{1}{1.6}}{\frac{1}{1.6} + \frac{1}{0.5}} \sim 19\%$$

with Puchalski,
PRL 101, 183001 (2008)

# Variational determination of the $Ps_2$ wave fnc.

Gaussian basis

$$\exp\left(-\sum_{i=1}^{6} a_i r_i^2\right)$$



Coordinate system for the positronium molecule

Relatively small basis ~ 2000
QR decomposition of the eigenvalue equation
Optimization of individual basis elements with Powell's method
→nonrelativistic energies ~1ppb
(test: Lithium)

Relativistic corrections dominated by annihilation: repulsive,
more effective in the ground state → decreases the SP interval.

# Summary

Methods of bound-state QED developed largely with positronium.

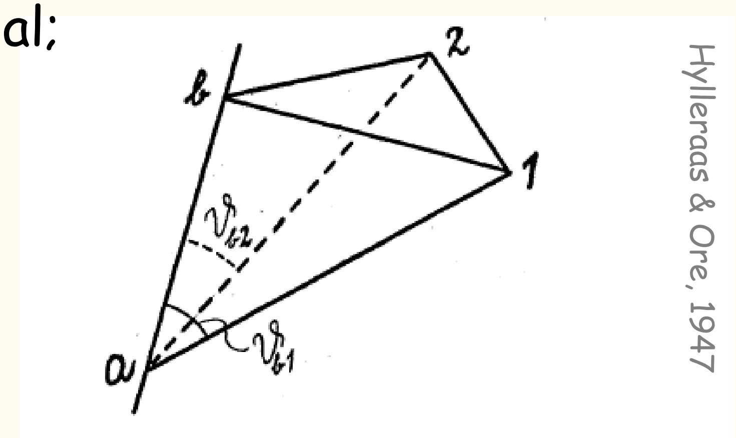
New states of positronium-like systems, ions and molecules: opportunity to test three- and four-body QED.

The Coulomb wave functions not known analytically for $e^+ e^- e^-$ and $e^+ e^- e^+ e^-$ .

Variational methods very accurate. O(alpha$^2$) corrections computed for the ion decay rate and the $Ps_2$ P-S interval. Both are being measured.

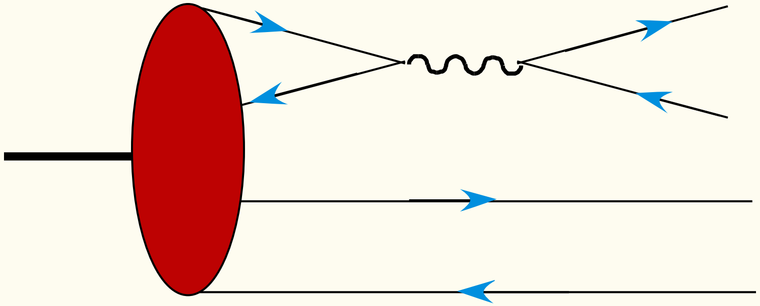
# Energy levels: ground state and P-excitation

Wave function determined variationally, M. Puchalski
using Coulomb potential;



Coordinate system for the positronium molecule

Relativistic corrections: perturbations.
Annihilation dominates.



Interval P-S determined with 5 x $10^{-6}$ accuracy
(slightly smaller than in Ps, "dielectric effect").

# Breakdown of corrections to the Ps ion width

| Correction | Value |
|---|---|
| $\alpha A^{3\gamma}$ | 0.002 693 245 |
| $\alpha A^{2\gamma}$ | −0.005 882 770 |
| $-2\alpha^2 \ln\alpha$ | 0.000 524 019 |
| $\alpha^2 B^{4\gamma}$ | 0.000 001 480 |
| $\alpha^2 B^{3\gamma}$ | −0.000 064 352 |
| $\alpha^2 B_{\text{squared}}$ | 0.000 008 652 |
| $\alpha^2 B^{\text{fin}}_{\text{hard}}$ | −0.000 218 3(34) |
| $\alpha^2 B_{\text{aa}}$ | 0.000 017 750 |
| $\alpha^2 B_{\text{H1}}$ | 0.000 078 366 |
| $\alpha^2 B_{\text{H2}}$ | 0.000 541 484 |
| $3\alpha^3 \ln^2\alpha/(2\pi)$ | −0.000 004 491 |
| $2.5(2.5)\alpha^3 \ln\alpha$ | −0.000 004 8(48) |
| Total $C$ | −0.002 309 7(59) |

$$\Gamma(\mathrm{Ps}^-) = 2\pi \frac{\alpha^5 m_e c^2}{\hbar}(1 + C)\left\langle \delta^3(r_{12})\right\rangle$$

# Breakdown of corrections to the $Ps_2$ interval

$$H_{\rm rel} = H_{\rm MV} + H_{\rm D} + H_{\rm OO} + H_{\rm SS} + H_{\rm A},$$

$$H_{\rm MV} = -\frac{1}{8}\sum_a \vec{p}_a^{\,4},$$

$$H_{\rm D} = -\pi \sum_{a<b} z_{ab}\delta^3(r_{ab}),$$

$$H_{\rm OO} = -\frac{1}{2}\sum_{a<b} z_{ab} p_a^i \left(\frac{\delta^{ij}}{r_{ab}} + \frac{r_{ab}^i r_{ab}^j}{r_{ab}^3}\right) p_b^j,$$

$$H_{\rm SS} = -\frac{2\pi}{3}\sum_{a<b} z_{ab}\vec{\sigma}_a \cdot \vec{\sigma}_b \delta^3(r_{ab}),$$

$$H_{\rm A} = \frac{\pi}{2}\sum_{a<b,ab\neq 12,34} (3 + \vec{\sigma}_a \cdot \vec{\sigma}_b)\delta^3(r_{ab}).$$

| Source | Ground state | $P$ state |
|---|---|---|
| $H_{\rm C}$ | −0.516 003 790 415(88) | −0.334 408 317 34(81) |
| $\alpha^2 H_{\rm MV}$ | −0.000 009 152 | −0.000 004 780(1) |
| $\alpha^2 H_{\rm OO}$ | −0.000 013 470 | −0.000 007 736 |
| $\alpha^2 H_{\rm D}$ | 0.000 014 592 | 0.000 007 458 |
| $\alpha^2 H_{\rm SS}$ | 0.000 000 419 | 0.000 000 097 |
| $\alpha^2 H_{\rm A}$ | 0.000 022 202 | 0.000 011 259 |
| $\alpha^2 H_{\rm rel}$ | 0.000 014 591 | 0.000 006 298(1) |
| $\alpha^3 \ln\alpha H_{\rm log}$ | 0.000 001 01(50) | 0.000 000 51(25) |
| Total | −0.515 988 2(5) | −0.334 401 5(3) |
| [12] | −0.515 989 199 656 | |

$$\Delta E \equiv E_P - E_S = 0.181\,586\,7(8)\ \text{a.u.}$$

# Ratio of three- to two-photon annihilation

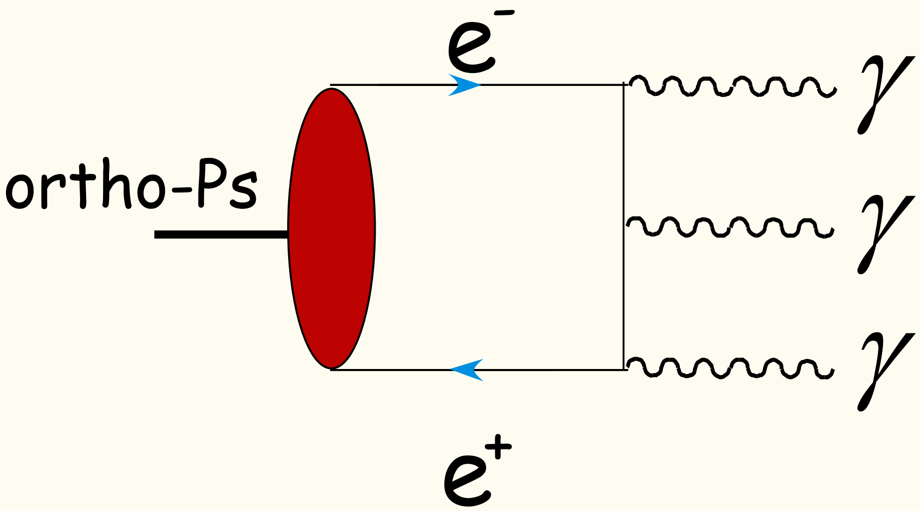
$$\mathrm{BR}\,(\mathrm{Ps}^- \longrightarrow \gamma\gamma\gamma) \equiv \frac{\Gamma(\mathrm{Ps}^- \longrightarrow \gamma\gamma\gamma)}{\Gamma(\mathrm{Ps}^-)}$$

$$= \alpha\left[A^{3\gamma} + \alpha(B^{3\gamma} - AA^{3\gamma}) - \frac{7}{3}A^{3\gamma}\alpha^2 \ln\frac{1}{\alpha} + \ldots\right]$$

$$= 0.002\,635\,8(8).$$

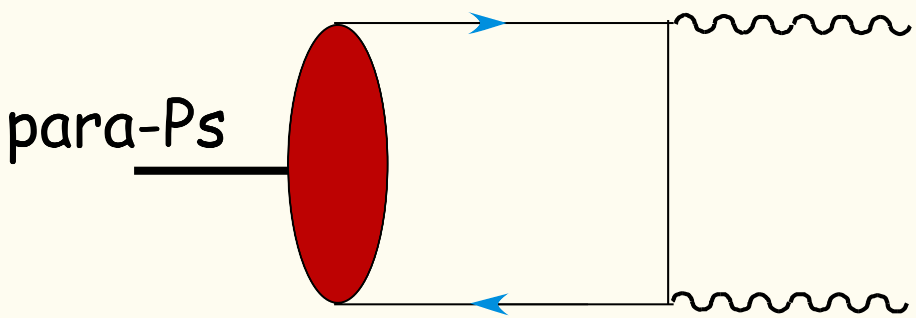
# Back to positronium...

## ... and its decays




$$\Gamma_{\exp}\left(\text{o-Ps}\right)=7.0401\left(7\right)\mu s^{-1}$$

$$1\cdot 10^{-4}$$




$$\Gamma_{\exp}\left(\text{p-Ps}\right)=7.9909\left(17\right)\text{ns}^{-1}$$

$$2\cdot 10^{-4}$$

# Theory of positronium decay

Corrections $O(\alpha)$

single hard photon loops

Corrections $O(\alpha^2)$

Para: AC, Melnikov, Yelkhovsky
Ortho: Adkins, Fell, Sapirstein

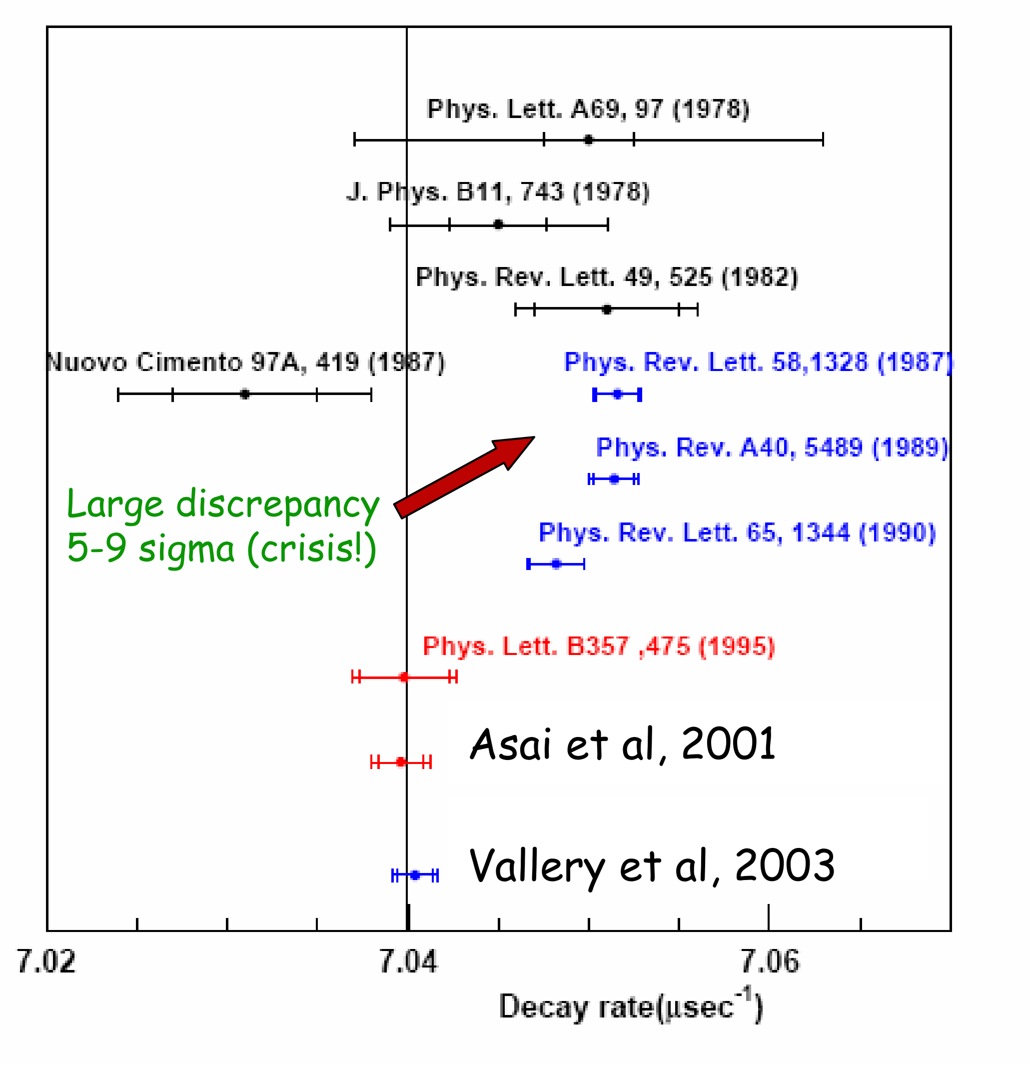
soft
- $O(k^2)$ corrections to decay amplitude
- Breit hamiltonian $\rightarrow$ correction to $\psi$ (r=0)

hard
- Short-distance two-loop photon exchange

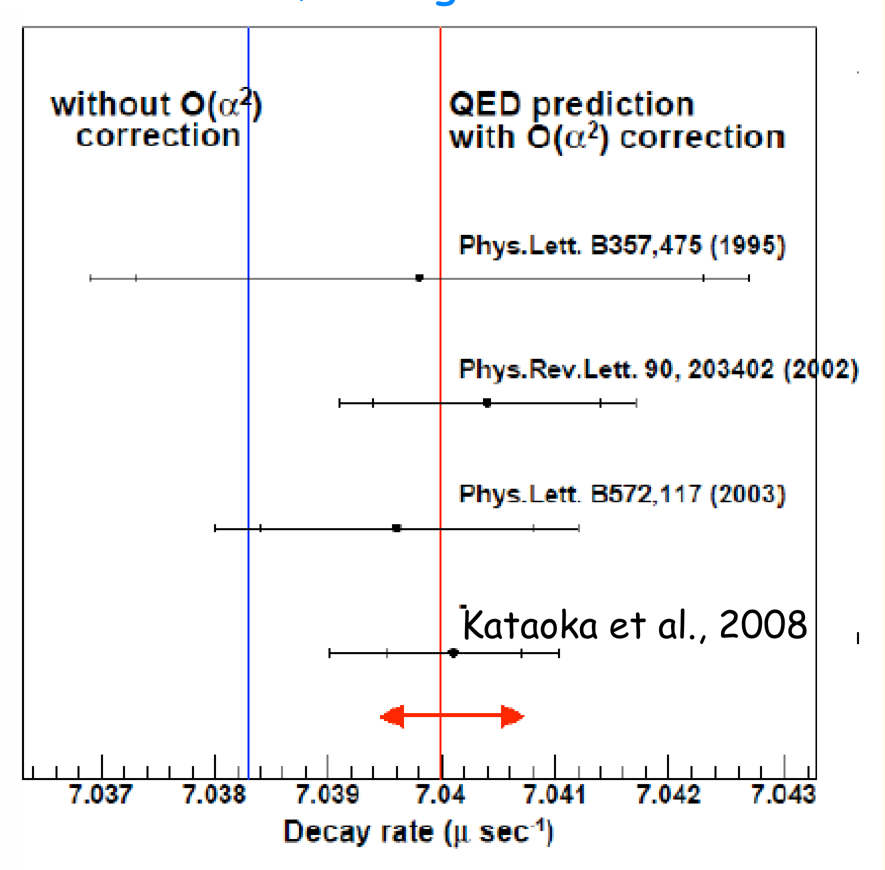
(last two items) finite together, but give $\ln \frac{m}{m\alpha} = \ln \frac{1}{\alpha}$

# History of oPs lifetime measurements

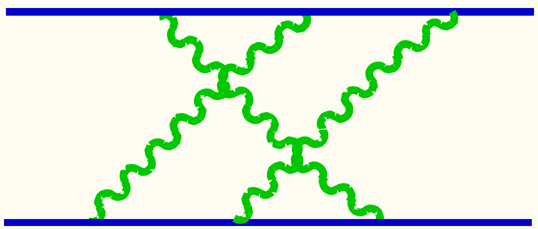
Older:




Latest ones, enlarged scale:



# Recoil effect and log(α)



Hard photons, k~$m_e$, generate a $\delta^3(r)$ potential. Its coefficient can be found from scattering of free electrons: it is IR divergent.

Conversely, soft photons, through the iterated Breit interaction, give a UV divergence.
The trick is to find a common regularization scheme for both.

Pineda and Soto suggested (1998) dimensional regularization,

$$\frac{(\alpha m)^{d-3}}{d-3} - \frac{m^{d-3}}{d-3} \xrightarrow{d\to 3} \ln\alpha$$